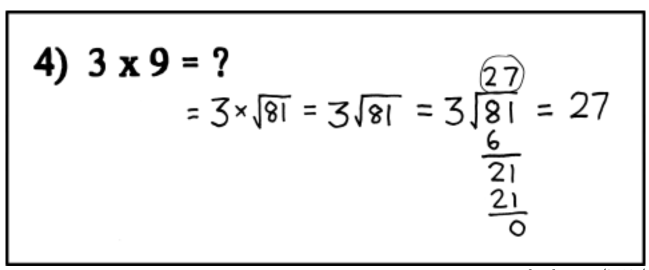
Physics GRE:

Summary

G. J. Loges[1]

University of Rochester
Dept. of Physics & Astronomy



xkcd.com/759/

Contents

1 Test-Taking

1.1 Strategies The Physics GRE subject test consist of 100 multiple-choice questions in 170 minutes. Each question has five choices; a correct response earns you one point and an incorrect response deducts a quarter of a point. Your raw score is converted to a scale in 10 point jumps out of 990, with a median score in the mid-six-hundreds. The test covers most aspects of an undergraduate physics curriculum. While it may not be the best indicator of your ability to do well in a physics graduate program, it does show that you have your foundations covered. Doing well will only help your chances of standing out.

- Move quickly! You have fewer than two minutes per question. Some questions you will know very quickly and others will take a good five minutes to finish the calculations. Mark involved questions and come back to them later. Set a timer for the practice tests and hold yourself to it!
- Have a bunch of common constants on quick recall such as $m_e = 0.5\,\mathrm{MeV}/c^2$ and $hc = 1240\,\mathrm{eV\,nm}$. Also, estimate like crazy when you can. If the five answers span many orders of magnitude chop everything down to one significant figure and go from there.
- Use dimensional analysis and limiting cases to narrow down the choices.
- Answer questions, even if you are not sure. Taking a random guess has an expected value of zero, and if you can eliminate one or more options then guessing is likely to be worthwhile.

1.2 Useful Quantities While there is a list of constants at the beginning of the test, it is useful to have a collection of common quantities available for quick use.

Name	Symbol	Approximate Value
Speed of light	c	$3.0 \cdot 10^8\,\mathrm{m/s}$
Electric charge	e	$1.6 \cdot 10^{-19}\,\mathrm{C}$
Electron mass	m_e	$0.5\,\mathrm{MeV}/c^2$
Proton mass	m_p	$940\,\mathrm{MeV}/c^2$
Proton/Electron mass ratio	m_p/m_e	1800
Hydrogen ground state energy	E_1	$-13.6\,\mathrm{eV}$
–	hc	$1240\,\mathrm{eV\,nm}$
Fine structure constant	$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$	$\frac{1}{137}$
Bohr radius	$a_0 = \frac{\hbar}{m_e c\alpha}$	$0.53\,\text{Å}$
Electron Compton wavelength	$\lambda_C = \frac{h}{m_e c}$	$0.0024\,\mathrm{nm}$
Visible spectrum wavelengths	–	$400 - 700\,\mathrm{nm}$
Room temperature thermal energy	kT_{room}	$\frac{1}{40}\,\mathrm{eV}$

1.3 Binomial Expansion For integral powers we may expand binomials as

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \tag{1}$$

β	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{\sqrt{3}}{2}$	$\frac{9}{10}$	$\frac{12}{13}$	$\frac{95}{100}$	$\frac{98}{100}$	$\frac{99}{100}$	$\frac{995}{1000}$	$\frac{999}{1000}$
	0	0.5	0.6	0.8	0.87	0.9	0.92	0.95	0.98	0.99	0.995	0.999
γ	1	$\frac{2}{\sqrt{3}}$	$\frac{5}{4}$	$\frac{5}{3}$	2	$\frac{10}{\sqrt{19}}$	$\frac{13}{5}$	$\frac{20}{\sqrt{39}}$	$\frac{50}{3\sqrt{11}}$	$\frac{100}{\sqrt{199}}$	$\frac{200}{\sqrt{399}}$	$\frac{1000}{\sqrt{1999}}$
	1	1.15	1.25	1.67	2	2.29	2.6	3.2	5.0	7.1	10.0	22.4

Table 1: Values of the Lorentz factor for several values of $\beta = \frac{v}{c}$.

This generalizes to nonintegral powers through the use of the gamma function for the binomial coefficients:

$$(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k y^{r-k} = y^r + rxy^{r-1} + \frac{r(r-1)}{2}x^2y^{r-2} + \frac{r(r-1)(r-2)}{6}x^3y^{r-3} + \cdots \tag{2}$$

However, due to the infinities the sum may not converge:

$$(1+x)^{-1} = 1 - x + x^2 - \cdots \qquad |x| < 1 \tag{3}$$

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots \qquad |x| < 1 \tag{4}$$

$$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \cdots \qquad |x| < 1 \tag{5}$$

These provide a quick way to estimate quantities such as those arising in doppler shift and special relativity topics.

2 Classical Mechanics

Kinematics

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \boldsymbol{v}_0 t + \frac{1}{2}\boldsymbol{a}t^2 \tag{Constant Acceleration}$$

$$\boldsymbol{v}(t) = \boldsymbol{v}_0 + \boldsymbol{a}t$$

$$y_{\max} = y_0 + \frac{v_{0y}^2}{2g} \tag{Max Height}$$

$$R = \frac{v^2}{g}\sin 2\theta \tag{Range Equation}$$

Newton's Laws

$$\sum \boldsymbol{F} = 0 \iff \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = 0 \tag{1st Law}$$

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\boldsymbol{v})}{\mathrm{d}t} \tag{2d Law}$$

$$\boldsymbol{F}_{\mathrm{AB}} = -\boldsymbol{F}_{\mathrm{BA}} \tag{3d Law}$$

Work & Energy

$$W = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} \tag{Work}$$

$$\sum W = \Delta E \tag{Work-Energy Theorem}$$
$$\boldsymbol{F} = -\boldsymbol{\nabla} U \tag{Conservative Force}$$
$$T = T_t + T_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{p^2}{2m} + \frac{L^2}{2I} \tag{Kinetic Energy}$$
$$f_\mathrm{s} \leq \mu_\mathrm{s} N \tag{Static Friction}$$
$$f_\mathrm{k} = \mu_\mathrm{k} N \tag{Kinetic Friction}$$
$$\boldsymbol{J} = \Delta \boldsymbol{p} = \int \boldsymbol{F}(t)\,\mathrm{d}t \tag{Impulse}$$

Rotational Motion

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{Constant Angular Acceleration}$$
$$\omega(t) = \omega_0 + \alpha t$$
$$T = \frac{1}{f} = \frac{2\pi}{\omega} \tag{Period $\leftrightarrow$ Frequency}$$
$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} \tag{Torque}$$
$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = I\boldsymbol{\omega} \tag{Angular Momentum}$$
$$I = \sum_i m_i r_i^2 = \int r^2\,\mathrm{d}m = \int r^2 \rho(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} \tag{Momentum of Inertia}$$
$$\boldsymbol{\tau} = \frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t} = \frac{\mathrm{d}(\boldsymbol{r} \times \boldsymbol{p})}{\mathrm{d}t} = \frac{\mathrm{d}(I\boldsymbol{\omega})}{\mathrm{d}t} \tag{Angular 2$^\mathrm{d}$ Law}$$

Noninertial Reference Frames

$$\boldsymbol{F}_\mathrm{cf} = -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \tag{Centrifugal Force}$$
$$\boldsymbol{F}_\mathrm{Co} = -2m\boldsymbol{\omega} \times \boldsymbol{v} \tag{Coriolis Force}$$

Dynamics of Systems of Particles

$$\boldsymbol{r}_\mathrm{cm} = \frac{1}{M}\sum_i m_i \boldsymbol{r}_i = \frac{1}{M}\int \boldsymbol{r}\,\mathrm{d}m \tag{Center of Mass}$$

Central Forces & Celestial Mechanics

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \text{const.} \tag{Kepler's 2$^\mathrm{d}$ Law}$$
$$\frac{T^2}{a^3} = \frac{4\pi^2}{G(M+m)} \approx \frac{4\pi^2}{GM} \tag{Kepler's 3$^\mathrm{d}$ Law}$$
$$g = \frac{GM_\oplus}{R_\oplus^2} \approx 9.81\,{}^\mathrm{N}/_\mathrm{kg} \tag{Earth Surface Gravity}$$

Lagrangian Mechanics

$$S = \int L(\boldsymbol{q}, \dot{\boldsymbol{q}}; t)\,\mathrm{d}t \tag{Action}$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\boldsymbol{q}}} - \frac{\partial L}{\partial \boldsymbol{q}} \qquad \text{(Euler-Lagrange Equations)}$$

Hamiltonian Mechanics

$$\boldsymbol{p} = \frac{\partial L}{\partial \dot{\boldsymbol{q}}} \qquad \text{(Conjugate Momentum)}$$

$$H(\boldsymbol{q}, \boldsymbol{p}; t) = \dot{\boldsymbol{q}} \cdot \boldsymbol{p} - L(\boldsymbol{q}, \dot{\boldsymbol{q}}; t) \qquad \text{(Hamiltonian)}$$

$$\dot{\boldsymbol{q}} = \frac{\partial H}{\partial \boldsymbol{p}} = \{\boldsymbol{q}, H\} \qquad \text{(Hamilton's Equations)}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{q}} = \{\boldsymbol{p}, H\}$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

Fluid Dynamics

$$P = \frac{\mathrm{d}F}{\mathrm{d}A} \qquad \text{(Pressure)}$$

$$\text{const.} = P + \frac{1}{2}\rho v^2 + \rho g h \qquad \text{(Bernoulli Equation)}$$

$$0 = \frac{\mathrm{d}\rho}{\mathrm{d}t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{u}) \qquad \text{(Continuity Equation)}$$

$$Av = \text{const.} \qquad \text{(Incompressible Tube Flow)}$$

$$F_{\text{buoy}} = \rho_{\text{fl}} V g \qquad \text{(Archimedes' Principle)}$$

3 Electromagnetism

Electrostatics

$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}} \qquad \text{(Coulomb's Law)}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\boldsymbol{F}}{q} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\boldsymbol{r} - \boldsymbol{r}_i|^2}\frac{\boldsymbol{r} - \boldsymbol{r}_i}{|\boldsymbol{r} - \boldsymbol{r}_i|} \qquad \text{(Electric Field)}$$

$$\boldsymbol{E} = -\boldsymbol{\nabla} V \qquad \text{(Electric Potential)}$$

$$V(\boldsymbol{r}) = -\int_{\infty}^{\boldsymbol{r}} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$$

Magnetostatics

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B} = I\boldsymbol{l} \times \boldsymbol{B} \qquad \text{(Magnetic Force)}$$

$$\omega = \frac{qB}{m} \qquad \text{(Cyclotron Frequency)}$$

$$\mathrm{d}\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2} \qquad \text{(Biot-Savart Law)}$$

Maxwell's Equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \qquad \text{(Gauss' Law)}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \qquad \text{(Gauss' Law for Magnetism)}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{(Faraday's Law)}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \qquad \text{(Ampère's Law)}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} = \epsilon \boldsymbol{E} = \epsilon_0 (1 + \chi_e) \boldsymbol{E} \qquad \text{(Auxillary Fields)}$$

$$\boldsymbol{H} = \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M} = \frac{1}{\mu} \boldsymbol{B} = \frac{1}{\mu_0 (1 + \chi_m)} \boldsymbol{B}$$

Electromagnetic Induction

$$\Phi_{\mathrm{E}} = \int_A \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{A} \qquad \text{(Electric Flux)}$$

$$\Phi_{\mathrm{B}} = \int_A \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A} \qquad \text{(Magnetic Flux)}$$

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{d}t} \qquad \text{(Induced EMF)}$$

$$L = \mu_0 n^2 A l \qquad \text{(Solenoid Self-Inductance)}$$

Electromagnetic Waves

$$\frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \nabla^2 \boldsymbol{E} \qquad \text{(Vacuum Wave Equations)}$$

$$\frac{1}{c^2} \frac{\partial^2 \boldsymbol{B}}{\partial t^2} = \nabla^2 \boldsymbol{B}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad \text{(Speed of Light)}$$

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{n} \qquad \text{(Speed of Light in Medium)}$$

Circuits

$$u = \frac{I}{nAq} \qquad \text{(Drift Velocity)}$$

$$V = IR \qquad \text{(Ohm's Law)}$$

$$P = IV = I^2 R = \frac{V^2}{R} \qquad \text{(Disspated Power)}$$

$$Q = CV \qquad \text{(Capacitance)}$$

$$U = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{Q^2}{2C} \qquad \text{(Energy of Capacitor)}$$

$$V = L \frac{\mathrm{d}I}{\mathrm{d}t} \qquad \text{(Inductor Potential)}$$

$$U = \frac{1}{2} L I^2 \qquad \text{(Energy of Inductor)}$$

$$R_{\text{ser}} = \sum_i R_i \tag{Effective Resistance}$$

$$\frac{1}{R_{\text{par}}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{C_{\text{ser}}} = \sum_i \frac{1}{C_i} \tag{Effective Capacitance}$$

$$C_{\text{par}} = \sum_i C_i$$

$$\oint_C V \, \mathrm{d}l = 0 \tag{Kirchhoff Loop Rule}$$

$$\sum_i I_i = 0 \tag{Kirchhoff Current Rule}$$

$$\tau = RC \tag{RC Time Constant}$$

$$\tau = \frac{R}{L} \tag{RL Time Constant}$$

$$\omega = \frac{1}{\sqrt{LC}} \tag{LC Frequency}$$

$$Z_{\text{C}} = \frac{1}{i\omega C} \tag{Capacitor Impedance}$$

$$Z_{\text{L}} = i\omega L \tag{Inductor Impedance}$$

4 Quantum Mechanics

$$p = \frac{h}{\lambda} \tag{de Broglie Wavelength}$$

$$K_{\text{max}} = h\nu - W = eV_{\text{stop}} \tag{Photoelectric Effect}$$

Operators

$$\langle A \rangle = \langle A | \Psi \rangle = \int \Psi^* A \psi \, \mathrm{d}\boldsymbol{r} \tag{Expected Value}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle A \rangle = \frac{1}{i\hbar} \langle [A, H] \rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle \tag{Ehrenfest Theorem}$$

$$p_i = -i\hbar \frac{\partial}{\partial x^i} \tag{Momemtum Operator}$$

$$T = \frac{\boldsymbol{p} \cdot \boldsymbol{p}}{2m} = -\frac{\hbar^2}{2m} \nabla^2 \tag{Kinetic Energy}$$

$$L_z = -i\hbar \frac{\partial}{\partial \phi} \tag{Angular Momentum}$$

$$H = T + V(x) = -\frac{\hbar^2}{2m} \nabla^2 + V(x) \tag{Nonrelativistic Hamiltonian}$$

$$a = \sqrt{\frac{m\omega}{2\hbar}} \left(x + \frac{ip}{m\omega} \right) = \sqrt{\frac{m\omega}{2\hbar}} \left(x + \frac{\hbar}{m\omega} \frac{\partial}{\partial x} \right) \tag{Harmonic Oscillator}$$

$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{ip}{m\omega} \right) = \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{\hbar}{m\omega} \frac{\partial}{\partial x} \right)$$

$$H = \hbar\omega\left(N + \frac{1}{2}\right) = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$[x_i, p_j] = i\hbar\delta_{ij} \qquad \text{(Cannonical coordinates)}$$

$$[f(x), p_x] = i\hbar f'(x) \qquad \text{(Function of position)}$$

$$[x, g(p_x)] = i\hbar g'(p_x) \qquad \text{(Function of momentum)}$$

$$[L_i, L_j] = i\hbar\epsilon_{ijk}L^k \qquad \text{(Angular momentum)}$$

$$[L^2, L_i] = 0$$

$$[a, a^\dagger] = 1 \qquad \text{(Ladder Operators)}$$

$$[N, a] = -a$$

$$[N, a^\dagger] = a^\dagger$$

One-Dimensional Potentials

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2} \qquad n \in \{1, 2, 3, \ldots\} \qquad \text{(Infinite square well)}$$

$$R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 \qquad T = \frac{4k_1k_2}{(k_1 + k_2)^2} \qquad \text{(Step: } E > V_0\text{)}$$

$$T = \left(1 + \frac{V_0^2\sin^2(k_2L)}{4E(E - V_0)}\right)^{-1} \qquad \text{(Finite Barrier: } E > V_0\text{)}$$

$$T = \left(1 + \frac{V_0^2\sinh^2(k_2L)}{4E(V_0 - E)}\right)^{-1} \qquad \text{(Finite Barrier: } E < V_0\text{)}$$

$$\psi(x) = \sqrt{\frac{m\beta}{\hbar^2}}\exp\left(-\frac{m\beta|x|}{\hbar^2}\right) \qquad V(x) = -\beta\delta(x) \qquad \text{(Delta Well Bound State)}$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\sqrt[4]{\frac{m\omega}{\pi\hbar}}H_n(\xi)e^{-\xi^2/2} \qquad \xi = \sqrt{\frac{m\omega}{\hbar}}x \qquad \text{(Harmonic Oscillator)}$$

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right) \qquad n \in \{0, 1, 2, \ldots\}$$

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right) \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{ip}{m\omega}\right)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}\left(a^\dagger + a\right) \qquad p = i\sqrt{\frac{\hbar m\omega}{2}}\left(a^\dagger - a\right)$$

Angular Momentum & Spin

$$L^2|l, m\rangle = \hbar^2 l(l+1)|l, m\rangle \qquad \text{(Eigenvalue Expressions)}$$

$$L_z|l, m\rangle = \hbar m|l, m\rangle$$

$$L_\pm|l, m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}|l, m\pm 1\rangle \qquad \text{(Ladder Operators)}$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \text{(Pauli Matrices)}$$

$$\sigma_i^2 = \mathbb{I}_2 \qquad \det\sigma_i = -1 \qquad \operatorname{tr}\sigma_i = 0$$

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij}\mathbb{I}_2 \qquad [\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma^k$$

$$\boldsymbol{A}\cdot\boldsymbol{B}=\frac{1}{2}\left(C^2-A^2-B^2\right) \qquad \boldsymbol{C}\equiv\boldsymbol{A}+\boldsymbol{B} \qquad \text{(The “Trick”)}$$

Time-Independent Non-Degenerate Perturbation Theory

$$E_n^{(1)}=\langle n^{(0)}|H'|n^{(0)}\rangle \qquad \text{(1}^{\text{st}}\text{-order energy correction)}$$

$$|n^{(1)}\rangle=\sum_{k\neq n}\frac{\langle k^{(0)}|H'|n^{(0)}\rangle}{E_n^{(0)}-E_k^{(0)}}\,|k^{(0)}\rangle \qquad \text{(1}^{\text{st}}\text{-order eigenstate correction)}$$

$$E_n^{(2)}=\sum_{k\neq n}\frac{|\,\langle k^{(0)}|H'|n^{(0)}\rangle\,|^2}{E_n^{(0)}-E_k^{(0)}} \qquad \text{(2}^{\text{d}}\text{-order energy correction)}$$

5 Thermodynamics & Statistical Mechanics

Laws of Thermodynamics

$$T_1=T_2\ \wedge\ T_2=T_3 \qquad\Longrightarrow\qquad T_1=T_3 \qquad \text{(1}^{\text{st}}\text{ Law)}$$

$$đQ=\mathrm{d}E+đW \qquad \text{(2}^{\text{d}}\text{ Law)}$$

$$\Delta S\geq 0 \qquad \text{(3}^{\text{d}}\text{ Law)}$$

$$T\to 0 \qquad\Longrightarrow\qquad S\to S_0 \qquad \text{(4}^{\text{th}}\text{ Law)}$$

Thermodynamic Processes

$$S=k\log\Omega \qquad \text{(Entropy)}$$

$$\mathrm{d}S=\frac{đQ}{T}$$

$$\beta=\frac{1}{kT}=\frac{\partial\log\Omega}{\partial E}=\frac{1}{k}\frac{\partial S}{\partial E} \qquad \text{(Temperature)}$$

$$S+S'=\text{maximal}\ \wedge\ T=T' \qquad \text{(Thermal Equilibrium)}$$

$$đQ=0 \qquad \text{(Adiabatic Processes)}$$

$$PV^{\gamma}=\text{const.}$$

$$TV^{\gamma-1}=\text{const.}$$

$$P^{1-\gamma}T^{\gamma}=\text{const.}$$

$$\mathrm{d}T=0 \qquad \text{(Isothermal Processes)}$$

$$W=nRT\log\left(\frac{V_2}{V_1}\right)$$

$$\mathrm{d}P=0 \qquad \text{(Isobaric Processes)}$$

$$W=P\Delta V$$

Ideal Gases

$$PV=nRT=NkT \qquad \text{(Ideal Gas Law)}$$

$$\mathrm{d}E=\left(\frac{\partial E}{\partial T}\right)_V\mathrm{d}T \qquad \text{(Ideal Gas Energy)}$$

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT=NkT \qquad \text{(Van der Waals Equation)}$$

Equipartition Theorem

$$\begin{aligned} E(q_1,q_2,\ldots,q_n) &= E_1(q_1)+E'(q_2,\ldots,q_n)=E_1+E' && \text{(Assumptions)}\\ E_1(q_1) &= Aq_1^r \\ \langle E_1\rangle &= \frac{1}{r}kT && \text{(Conclusion)} \end{aligned}$$

Maxwell Relations

$$\begin{aligned} \left(\frac{\partial T}{\partial V}\right)_S &= -\left(\frac{\partial P}{\partial S}\right)_V & \left(\frac{\partial T}{\partial P}\right)_S &= \left(\frac{\partial V}{\partial S}\right)_P && \text{(Maxwell Relations)}\\ \left(\frac{\partial S}{\partial V}\right)_T &= \left(\frac{\partial P}{\partial T}\right)_V & \left(\frac{\partial S}{\partial P}\right)_T &= -\left(\frac{\partial V}{\partial T}\right)_P \end{aligned}$$

$$\begin{aligned} H &= E+PV && \text{(Enthalpy)}\\ G &= H-TS && \text{(Gibb's Free Energy)}\\ F &= E-TS && \text{(Helmholtz Free Energy)} \end{aligned}$$

$-S$	U	V
H		F
$-P$	G	T

(Mnemonic)

Statistical Mechanics

$$\begin{aligned} Z &= \sum_i e^{-\beta E_i} = \sum_E \Omega(E)e^{-\beta E} && \text{(Partition Function)}\\ Z &= \prod_i \zeta_i && \text{(Weakly Interacting Subsystems)}\\ Z &= \frac{\zeta^N}{N!} && \text{(Indistinguishable Subsystems)}\\ \langle E\rangle &= -\frac{\partial \log Z}{\partial \beta} && \text{(Average Energy)}\\ S &= k(\log Z+\beta\langle E\rangle) && \text{(Entropy)}\\ C_{\mathrm{V}} &= \frac{1}{kT^2}\frac{\partial^2 \log Z}{\partial T^2} && \text{(Specific Heat)} \end{aligned}$$

Particle Statistics

$$\begin{aligned} \langle n_i\rangle &= N\cdot\frac{e^{-\beta E_i}}{Z} && \text{(Maxwell-Boltzmann Statistics)}\\ \langle n_i\rangle &= \frac{1}{e^{(E_i-\mu)/kT}-1} && \text{(Bose-Einstein Statistics)}\\ \langle n_i\rangle &= \frac{1}{e^{(E_i-\mu)/kT}+1} && \text{(Fermi-Dirac Statistics)}\\ \langle n_i\rangle &= \frac{1}{e^{E_i/kT}-1} && \text{(Photon Statistics)} \end{aligned}$$

Theories of Specific Heat

$$
\begin{aligned}
c_\mathrm{V} &= 3N_\mathrm{A}k\left(\frac{h\nu}{kT}\right)^2\frac{e^{h\nu/kT}}{\left(e^{h\nu/kT}-1\right)^2} && \text{(Einstein Theory)}\\
c_\mathrm{V} &\to \alpha T^3 && \text{(Debye Low Temperature)}\\
c_\mathrm{V} &\to 3N_\mathrm{A}k = 3R && \text{(Einstein \& Debye High Temperature)}
\end{aligned}
\tag{6}
$$

6 Atomic Physics

Bohr Model

$$
\begin{aligned}
L &= n\hbar \qquad n \in \{1,2,3,\ldots\} && \text{(Quantization Assumption)}\\
r_n &= \frac{\hbar^2 n^2}{Zke^2\mu} = \frac{a_0 n^2}{Z} && \text{(Radii)}\\
a_0 &= \frac{\hbar^2}{kme^2} \approx 0.529\,\text{Å} && \text{(Bohr Radius)}\\
E_n &= -\frac{Z^2k^2e^4\mu}{2\hbar^2n^2} = -13.6\,\mathrm{eV}\frac{Z^2}{n^2} && \text{(Hydrogen Energy Levels)}\\
n &\in \{1,2,3,\ldots\} && \text{(Principle Quantum Nº)}\\
l &\in \{0,1,\ldots,n-1\} && \text{(Azimuthal Quantum Nº)}\\
m_l &\in \{-l,-l+1,\ldots,l\} && \text{(Magnetic Quantum Nº)}\\
m_s &\in \{-s,-s+1,\ldots,s\} && \text{(Spin Projection Quantum Nº)}
\end{aligned}
$$

Selection Rules

$$
\begin{aligned}
\Delta l &= \pm 1 && \text{(Electric Dipole Selection Rules)}\\
\Delta m &= 0, \pm 1\\
\Delta s &= 0\\
j_i &= 0 \not\to j_f = 0
\end{aligned}
$$

Blackbody Radiation

$$
\begin{aligned}
I_\mathrm{P}(\lambda,T) &= \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} && \text{(Planck's Law)}\\
\lambda_\mathrm{peak}T &= 2.9\cdot 10^6\,\mathrm{nm\,K} && \text{(Wien's Displacement Law)}\\
I_\mathrm{RJ}(\lambda,T) &= \frac{2ckT}{\lambda^4} && \text{(Raleigh-Jeans Law)}\\
I_\mathrm{W} &= \frac{2hc^2}{\lambda^5}e^{-\frac{hc}{\lambda kT}} && \text{(Wien's Law)}
\end{aligned}
$$

Compton Scattering

$$
\Delta\lambda = \frac{h}{mc}(1-\cos\theta) \qquad \text{(Compton Shift)}
$$

$$\lambda_{\mathrm{C}} = \frac{h}{mc} \qquad \text{(Compton Wavelength)}$$

$$\lambda_{\mathrm{C}} = \frac{h}{m_e c} \approx 2.43\,\mathrm{pm} \qquad \text{(Electron Compton Wavelength)}$$

Moseley's Law

$$E = 13.6\,\mathrm{eV}\left(\frac{3}{4}\right)(Z-1)^2 \qquad (K\alpha \text{ Photons: } n = 2 \to 1)$$

$$E = 13.6\,\mathrm{eV}\left(\frac{5}{36}\right)(Z-7.4)^2 \qquad (L\alpha \text{ Photons: } n = 3 \to 2)$$

7 Optics & Wave Phenomena

Wave Properties

$$\frac{1}{v_{\mathrm{p}}^2}\frac{\partial^2\phi}{\partial t^2} = \nabla^2\phi \qquad \text{(Wave Equation)}$$

$$v_{\mathrm{p}} = \frac{\omega}{k} = \frac{\lambda}{T} \qquad \text{(Phase Velocity)}$$

$$v_{\mathrm{g}} = \frac{\mathrm{d}\omega(k)}{\mathrm{d}k} \qquad \text{(Group Velocity)}$$

$$2\pi f_{\mathrm{B}} = \omega_{\mathrm{B}} = |\omega_1 - \omega_2| \qquad \text{(Beat Frequency)}$$

Interference & Diffraction

$$n\lambda = d\sin\theta \approx d\cdot\frac{y}{D} \qquad \text{(Double-Slit Maxima)}$$

$$\left(n+\tfrac{1}{2}\right)\lambda = d\sin\theta \approx d\cdot\frac{y}{D} \qquad \text{(Double-Slit Minima)}$$

$$\left(n+\tfrac{1}{2}\right)\lambda = a\sin\theta \approx a\cdot\frac{y}{D} \qquad \text{(Single-Slit Maxima)}$$

$$n\lambda = a\sin\theta \approx a\cdot\frac{y}{D} \qquad \text{(Single-Slit Minima)}$$

$$I = I_0\cos^2\left(\frac{\pi d\,y}{\lambda D}\right)\mathrm{sinc}^2\left(\frac{\pi a\,y}{\lambda D}\right) \qquad \text{(Double-Slit Intensity)}$$

$$f_n = \frac{c}{\lambda_n} = \frac{(2n+1)c}{4L} = (2n+1)f_0 \qquad \text{(Half-Open Frequencies)}$$

$$f_n = \frac{c}{\lambda_n} = \frac{nc}{2L} = nf_1 \qquad \text{(Both Open Freqencies)}$$

Geometrical Optics

$$n_1\sin\theta_1 = n_2\sin\theta_2 \qquad \text{(Snell's Law)}$$

$$\theta_{\mathrm{crit}} = \arcsin\left(\frac{n_2}{n_1}\right) \qquad \text{(Critical Angle)}$$

$$\theta_{\mathrm{Brew}} = \arctan\left(\frac{n_2}{n_1}\right) \qquad \text{(Brewster's Angle)}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{(Lensmaker's Equation)}$$

$$\frac{1}{o} + \frac{1}{f} = \frac{1}{i} \qquad \text{(Thin Lens Equation)}$$

$$M = \frac{i}{o} \qquad \text{(Magnification)}$$

Doppler Effect

$$f = \left(\frac{c + v_\mathrm{o}}{c + v_\mathrm{s}}\right) f_0 \qquad \text{(Doppler Shift)}$$

$$f = \left(1 + \frac{\Delta v}{c}\right) f_0 \qquad \text{(Low Velocity Approximation)}$$

8 Relativity

Four-Vectors & Lorentz Transformations

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z) \qquad \text{(Four-Position)}$$

$$\eta_{\mu\nu} = \eta^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1) \qquad \text{(Minkowski Metric)}$$

$$M_\mu = \eta_{\mu\nu} M^\nu \qquad M^\mu = \eta^{\mu\nu} M_\nu \qquad \text{(Raising \& Lowering of Indices)}$$

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu = c^2 \mathrm{d}t^2 - \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) = c^2 \mathrm{d}t^2 - \mathrm{d}\boldsymbol{x}^2 \qquad \text{(Spacetime Interval)}$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \beta^2}} \qquad \text{(Lorentz Factor)}$$

$$\begin{bmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} \qquad \text{(Boost Along } x\text{-Axis)}$$

$$v' = \frac{v + u}{1 + \frac{vu}{c^2}} \qquad \text{(Colinear Velocity Addition)}$$

$$\boldsymbol{p} = \gamma m \boldsymbol{v} \qquad \text{(Momentum)}$$

$$E = \gamma m c^2 \qquad \text{(Energy)}$$

$$E^2 = (mc^2)^2 + |\boldsymbol{p}|^2 c^2$$

$$p^\mu = (p^0, p^1, p^2, p^3) = (\tfrac{E}{c}, p_x, p_y, p_z) \qquad \text{(Four-Momentum)}$$

$$p_\mu p^\mu = m^2 c^2$$

Time Dilation & Length Contraction

$$\Delta t = \gamma \Delta\tau \qquad \text{(Time Dilation)}$$

$$L = \frac{L_0}{\gamma} \qquad \text{(Length Contraction)}$$

Electromagnetism

$$
\begin{aligned}
A^\mu &= (A^0, A^1, A^2, A^3) = (\tfrac{V}{c}, A_x, A_y, A_z) && \text{(Four-Potential)}\\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu && \text{(Electromagnetic Field Tensor)}\\
E'_x &= E_x \qquad\qquad B'_x = B_x && \text{(Boost Along } x\text{-Axis)}\\
E'_y &= \gamma(E_y - vB_z) \qquad B'_y = \gamma(B_y + vE_z/c^2) &&\\
E'_z &= \gamma(E_z + vB_y) \qquad B'_z = \gamma(B_z - vE_y/c^2) &&
\end{aligned}
$$

Doppler Effect

$$\lambda = \frac{c}{f} = \lambda_0 \sqrt{\frac{1 \pm \beta}{1 \mp \beta}} = \frac{c}{f_0}\sqrt{\frac{1 \pm \beta}{1 \mp \beta}} \qquad \text{(Doppler Effect)}$$

General Relativity

$$r_\mathrm{s} = \frac{2GM}{c^2} \qquad \text{(Schwarzchild Radius)}$$

9 Specialized Topics

Radioactive Decay

$$
\begin{aligned}
\frac{\mathrm{d}N}{\mathrm{d}t} &= -\gamma N && \text{(Exponential Decay)}\\
N(t) &= N_0 e^{-\gamma t} = N_0 e^{-t/\tau} = N_0 2^{-t/t_{1/2}} &&\\
\tau &= \frac{1}{\gamma} = \frac{t_{1/2}}{\log 2} &&
\end{aligned}
$$

X-Ray Diffraction

$$2d\sin\theta = n\lambda \qquad n \in \{1, 2, 3, \ldots\} \qquad \text{(Bragg Diffraction)}$$

Vector Differential Operators

$$
\begin{aligned}
\boldsymbol{\nabla} f &= \frac{\partial f}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{k}} && \text{(Gradient)}\\
\boldsymbol{\nabla}\cdot\boldsymbol{F} &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} && \text{(Divergence)}\\
\boldsymbol{\nabla}\times\boldsymbol{F} &= \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\boldsymbol{\imath}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\boldsymbol{\jmath}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\boldsymbol{k}} && \text{(Curl)}\\
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} && \text{(Laplacian)}
\end{aligned}
$$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{Fourier Series}$$

$$a_m = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos mx \, \mathrm{d}x \qquad m \in \{0, 1, 2, \dots\}$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin kx \, \mathrm{d}x \qquad k \in \{1, 2, 3, \dots\}$$

Matrix Algebra

$$\operatorname{tr} A = \sum_i a_{ii} = \sum_i \lambda_i \tag{Trace}$$

$$\det A = \prod_i \lambda_i \tag{Determinant}$$

$$\det A_{2\times 2} = a_{11}a_{22} - a_{12}a_{21} \tag{2×2 Determinant}$$

$$A_{2\times 2}^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \tag{2×2 Inverse}$$

$$\lambda_{\pm} = \frac{\operatorname{tr} A}{2} \pm \sqrt{\left(\frac{\operatorname{tr} A}{2}\right)^2 - \det A} \tag{2×2 Eigenvalues}$$